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Using Helmert's Second Method of Condensation To Calculate the Indirect Effect to Geoid Undulation Over Egypt Territory

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Abstract

Whilst there are many factors posing hurdles in an accurate geoid model determination, only the effect of variable topographic mass density to the geoid undulation is of concern in this paper. Thus, the surface computed by Stokes formula from topographic-isostatics gravity anomalies is not the geoid itself but a slightly different, for every gravity reduction there corresponding a different co-geoid. Helmert's second method of condensation is used to calculate the indirect effect to geoid undulation over Egypt territory were the indirect effect range from statistical calculations using the first approximation of helmert condensations was centimeter up to 3 cm, all over whole territory of EGYPT except one only region is "SINAI" has been reached to 13 cm, were the orthometric heights are maximum. According to the topographic features of Egypt were almost flat except "SINAI" area where the huge mountains are existing, therefore the indirect effect has outstanding extreme value 13 cm at maximum orthometric height 1527 m.

Keywords: Indirect effect, topographic masses effect, terrain correction, geoid determination, co-geoid.

1. Introduction

A basic quantity that describes the Earth gravity field is the gravity potential W. this quantity could be solved in and outside the geoid were the geoid is the equipotential surface that approximates the mean sea level most closely. The gravity potential on the geoid is denoted by $W_{\circ}$= const. to solve this problem a normal gravity potential U generated by the mean geocentric ellipsoid of revolution is introduced. The normal gravity potential $U_{\circ}$ on the mean geocentric ellipsoid is chosen to be equal to the earths potential on the geoid $W_{\circ} = U_{\circ}$.

The difference of the gravity potential W and the normal potential U defines the disturbance potential T

$$T = W - U \quad (1)$$

When the atmospheric attraction is neglected, T is harmonic outside the earth and satisfy Laplace equation

$$\nabla^2 T = 0 \quad (2)$$

Once T has been solved, the gravity potential W can be obtained at any point by adding the normal potential U which can be computed from existing models.

Also, when T is known on the geoid, the geoid undulation can be obtained by Bruns formula [4]

$$N = \frac{T}{\gamma_{\circ}} \quad (3)$$

Where T is disturbing potential on the geoid, and $\gamma_{\circ}$ is the normal gravity on the mean geocentric ellipsoid. The problem now is to produce the disturbing potential T on and outside the geoid. However, the disturbing potential T does not satisfy the Laplace equation inside the topographic masses where the geoid often located. Therefore, in order to satisfy the Laplace equation, all atmospheric and topographical masses must be removed or condensed on or beneath the geoid. Many reduction methods have been suggested to minimize the indirect effect of reduction as shown in (Fig. 1), and one of the most frequently used is a Helmert's second condensation method were, the atmospheric and topographical masses are condensed directly onto the geoid [7].

The space obtained after such a condensation is the Helmert space. The quantities are given in the Helmert space denoted by superscript H this obtained by condensation of the atmospheric and topographic masses as a single layer that located on the geoid which causes slightly change in earth's gravity field.

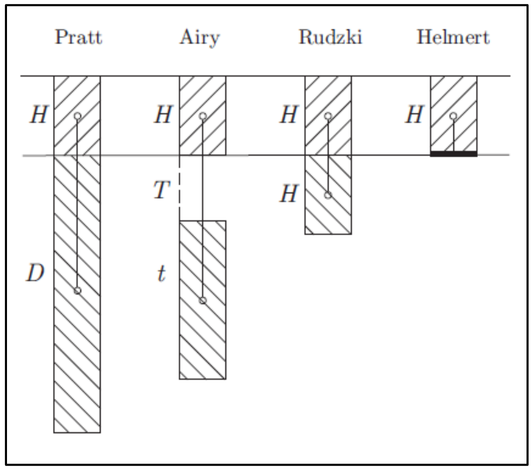

Fig. 1. Topography and compensation for different gravity reductions

HELMERT'S SECOND METHOD OF CONDENSATION

Helmert's second method of condensation can be described in three steps:

- Removal of Bouguer plate and determining gravity values on the physical surface of the Earth after subtracting the effect of removed masses,
- Applying a free air reduction in order to determine gravity value on the geoid,
- Condensation of previously removed masses of the Bouguer plate on the geoid itself.

After applying all three mentioned steps gravity value at the geoid obtained by Helmert's reduction can be determined by the equation:

$$g_H = g_P + \delta_{FA} \quad (4)$$

Where g_P is the gravity value measured on the physical surface of the Earth and δ_{FA} is the free air correction [4].

In addition, in order to adequately include the gravity effect of the Earth's atmosphere, atmospheric correction δ_A has to be added to measured gravity (Fig. 2):

$$g_H = g_P + \delta_{FA} + \delta_A \quad (5)$$

Subtracting the value of normal gravity at the surface of the ellipsoid from previously determined g_H it is possible to determine Helmert's anomalies [5], [6]:

$$\Delta g_H = g_P - \gamma_Q \quad (6)$$

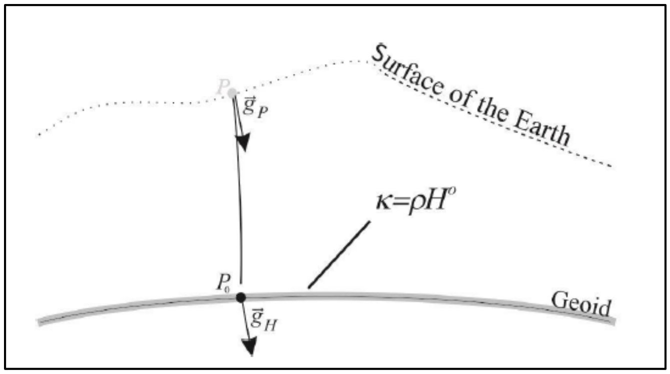

Fig. 2. Helmert's second method of condensation

This kind of anomalies we can apply in geoid undulation determination by the well-known Stokes formula [5]:

$$N = \frac{R}{4\pi\gamma} \iint_\sigma \Delta g \, S(\Psi) \, d\sigma \quad (7)$$

Where N is geoid undulation, R is the radius of the terrestrial sphere, σ is the unit sphere introduced for integration over the close surfaces and $S(\Psi)$ is the Stokes function with spherical distance Ψ .

Instead of undulation N from the (Equ.7), with applying Δg_H only some co-undulation N_c can be determined which is related to the actual N by the equation:

$$N = N^c + \delta N^c \quad (8)$$

Where δN^c is the indirect effect of reduction to the geoid undulation (Fig. 3).

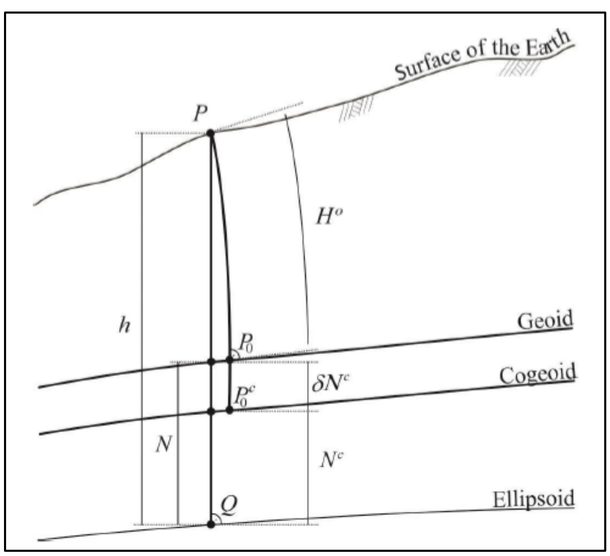

Fig. 3. Geoid and co-geoid

The first approximation of δN^c can be obtained by

$$\delta N^c = -\frac{\pi G \rho H^{\circ 2}}{\gamma_Q} \quad (9)$$

Where G is universal gravitational constant, ρ is the density of Earth's crust masses, H° is the orthometric height of evaluating point.

2. Available data and numerical investigation

As shown above in (Eq. 6) all right terms are constant except the orthometric heights if we consider the adopted value of density ρ is equal to 2670kg/m^3, therefore the only variable remaining is the orthometric height H, it is obvious that indirect effect, or at least their first approximation, can be determined on the whole territory of Egypt if it is possible to provide points with known orthometric heights. In order to numerically investigate the indirect effects of the second Helmert's reduction from a whole set of more than 3726 gravimetric points as shown in (Fig. 4).

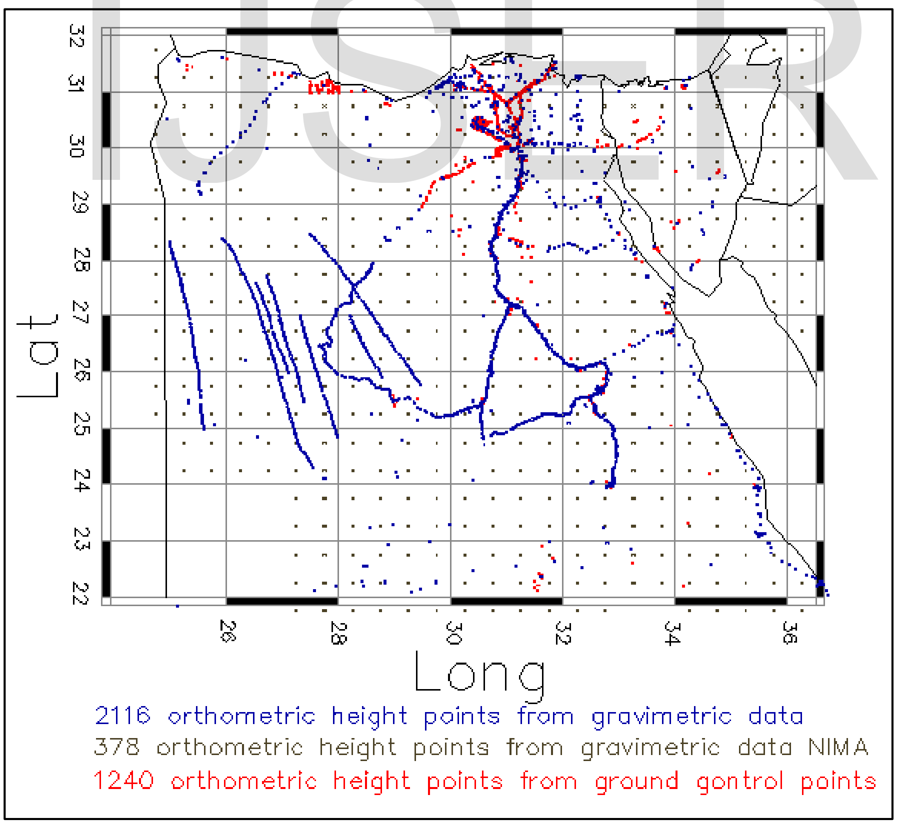

Fig. 4. The available data

Fig. [3] Shows an irregular distribution orthometric heights data with large gaps, especially on the northern desert, while Fig. [5] Illustrate the topographic features of Egypt. The local gravimetric data have been used in this study were grouped into three sets as shown in Fig. [4]. Firstly, all old available free-air gravity anomalies at 1849 points, where the sources of these data their number and distribution are well documented in many previous works as shown in [1], [2], [3] and gravimetric data values at 267 points were obtained from BGI [Bureau Gravimetric International], thirdly 378 gravimetric data points from NIMA terrestrial data in addition to local ground control point (GCP) at 1240 points were gathered from different EGYPTIAN construction projects.

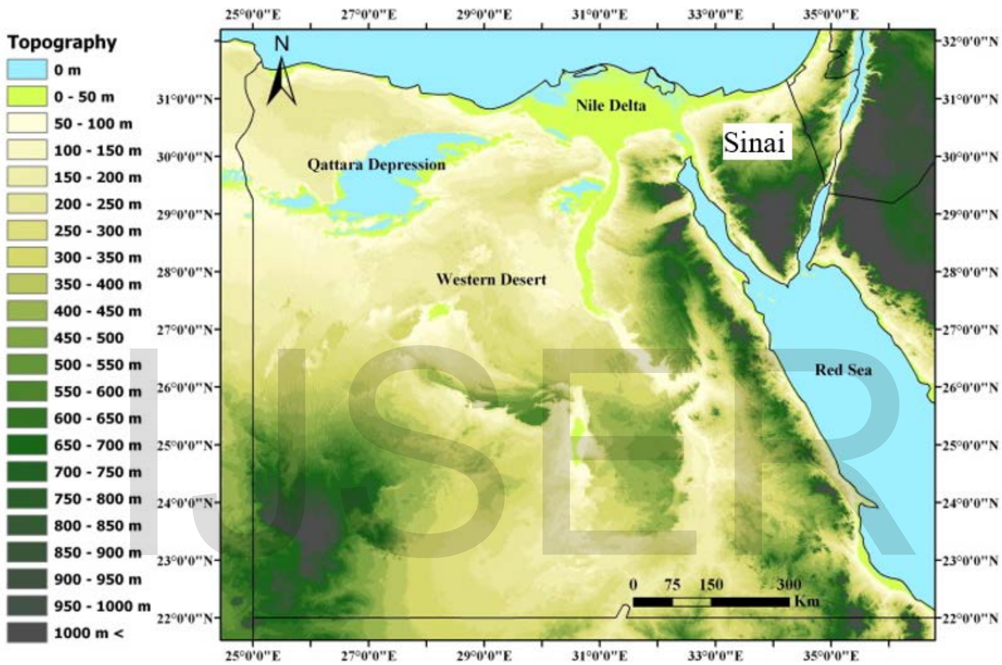

Fig. 5. The topographic features of Egypt territory

3. Statistical calculations and Results

Table 1: Basic statistical data of orthometric heights used in indirect effects determination over Egypt territory (m)

Parameter	Number of data points	Minimum (m)	Maximum (m)	Average (m)	Standard Deviation (m)
H°	3726	-6.000	1526.929	139.544	163.096

Table 2: Basic statistical data of indirect effects over Egypt territory (cm)

Parameter	Number of data points	Minimum (cm)	Maximum (cm)	Average (cm)	Standard Deviation (cm)
δN^c	3726	-13.317	0.000	-0.263	0.752

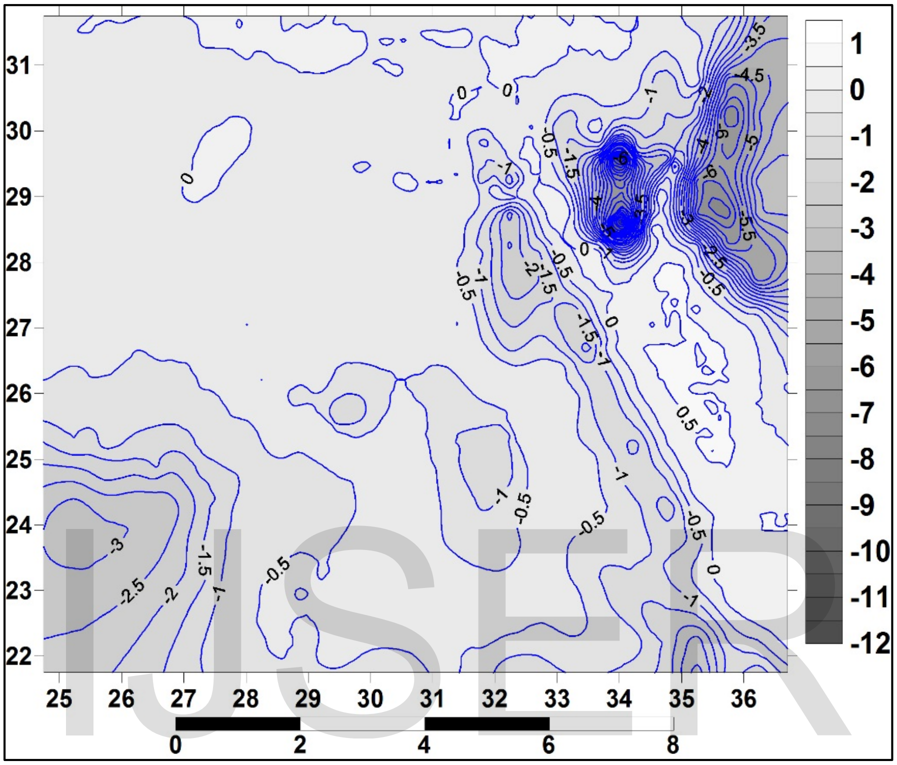

Fig. 6. General shape of the surface of the indirect effect to the geoid undulation in (cm)

The general shape of the indirect effect to the geoid undulation by applying the (eq. 6) to 3726 data points all over Egypt territory were illustrated in (Fig. 6) in addition to the numerical values which listed in the table. 2.

4. Conclusion

From (Fig. 6) and Table (2), it is obvious to conclude that the indirect effect range between sub-centimeter up to 3 cm approximately all over the whole territory of Egypt except only one region “SINAI”, where orthometric heights are maximum.

According to the topographic features of Egypt were almost flat except “SINAI” area where the huge mountains are existing, therefore the indirect effect has outstanding extreme value 13 cm at maximum orthometric height 1527 m as mentioned in table (3, 4).

Depending on the output results from this research is recommended to use the surface computed by Stokes formula as geoid its self in all territory of Egypt was the indirect effect to the geoid undulation is insignificant except “SINAI” it could subtract about 13 cm to reach its optimum geoid values.
Finally, at the constant value of the density of topographic masses, the estimated value of the indirect effect ranges from several millimeters to several centimeters, so it is important to emphasize that when determining the geoid of centimeter accuracy, information on the density of topographic masses should be included not only in determining the indirect effect but also in the all necessary steps of the geoid determination process.

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